

Gravitational Energy Handout

1. Why is g different at poles than at equator?
because

a) Earth is not a perfect sphere, it bulges in the middle



$$r_e > r_p$$

$$g = \frac{GM}{r^2}$$

$$g_e < g_p$$

b) Earth's spin results in masses on the equator having a slightly lower apparent weight due to the centripetal acceleration.

$$F_N = F_g - F_c$$

you are slightly in freefall.

both factors cause an effect of about 0.03 each

so equator 9.781 and pole is 9.832

about 0.05 N/kg difference

$$2. \ g = a = 4\pi^2 r / T^2 = 4\pi^2 (3.43 \times 10^6) / (102 \times 60)^2 = 3.62 \text{ N/kg}$$

$$3. \ a = 4\pi^2 r / T^2 = 4\pi^2 (3.9 \times 10^8) / (27.3 \times 24 \times 3600)^2 = 2.7 \times 10^{-3} \text{ m/s}^2$$

$$4. \ r = 6.38 \times 10^6 \text{ (height is negligible)}$$

$$a = g$$

$$v^2/r = GM/r^2 = 9.80 \text{ m/s}^2$$

$$v = \sqrt{9.8 \times 6.38 \times 10^6} = 7,907.211898008046$$

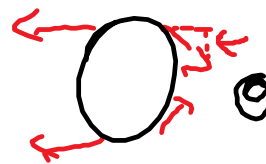
$$7.9 \times 10^3 \text{ m/s}$$

$$5. a = 4\pi^2 r / T^2 = 4\pi^2 (1.5 \times 10^{11}) / (365.25 \times 24 \times 3600)^2 = 5.95 \times 10^{-3} \text{ m/s}^2$$

6. Earth to Sun is about double Moon to Earth
Why does the Moon cause tides more strongly than the Sun?

Tides are caused by the tangential components of the gravity not the net gravity.

Since the Moon is closer, it has larger lateral component.



$$7. \cancel{m}a = \cancel{m}g$$

a) same b) $v^2/r = GM/r^2$ same c) same d) double
 $F_c = ma$ a is same, m is double so F_c is double

$$8. E_g = -GMm/r = -6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 1 / 6.38 \times 10^6 = -6.2518 \times 10^7 = -6.25 \times 10^7 \text{ J}$$

9. Due to the Earth's gravitational pull

$$E_{gf} = -6.67 \times 10^{-11} \times 5.98 \times 10^{24} \times 1 / 3.9 \times 10^8 = -1.0227 \times 10^6$$

$$E_{gf} - E_{gi} = -1.0227 \times 10^6 - (-6.2518 \times 10^7) = 61,495,300$$

$$6.1 \times 10^7 \text{ J}$$

keepers: include the change in gravitational energy due to the moon's pull:

$$-GMm(1/r_f - 1/r_i)$$

$$-6.67E-11 \times 7.35E22 \times 1 \times ((1/1.74E6) - (1/3.8E8)) = -2.8046E6$$

$$\text{so total is } 61,495,300 - 2.8046E6 = 58,690,700$$

$$5.9 \times 10^7 \text{ J}$$

Q10 escape velocity total energy is 0

$$E_g + E_k = 0$$

$$-GMm/r + 1/2mv^2 = 0$$

$$v = \sqrt{2GM/r} =$$

$$\sqrt{2 \times 6.67E-11 \times 7.35E22 / 1.74E6} =$$

$$2,373.815494093844$$

$$2.4 \text{ km/s}$$

Q11 $-GMm/r$ for Earth and Moon

Earth

$$-6.67E-11 \times 5.98E24 \times 1000 / 6.38E6 = -6.2518E10$$

Moon

$$-6.67E-11 \times 7.35E22 \times 1000 / 1.74E6 = -2.8175E9$$

$$6.3 \times 10^{10} \text{ J vs } 2.8 \times 10^9 \text{ J}$$

$$\text{way less energy } -2.8175E9 / -6.2518E10 = 0.0451$$

about 1/20th of the energy required not including air resistance!

Any questions about the quiz next class?

eg. The moon has a mass of $7.35 \times 10^{22} \text{kg}$ and a radius of $1.77 \times 10^6 \text{m}$. If you are in orbit around the moon at a height of 400km, what is your (if your mass is 50.0kg)

a) gravitational energy in that orbit relative to 0 at infinity?

$$E_g = -GMm/r$$

$$= -6.67 \times 10^{-11} \times 7.35 \times 10^{22} \times 50 / (1.77 \times 10^6 + 400000) = -1.1296 \times 10^8$$

$$-1.13 \times 10^8 \text{J}$$

b) kinetic energy in that orbit?

$$E_k = 1/2 |E_g| = 0.5 \times 1.1296 \times 10^8 = 56,480,000$$

$$5.65 \times 10^7 \text{J}$$

$$v^2/r = GM/r^2$$

$$v = \sqrt{6.67 \times 10^{-11} \times 7.35 \times 10^{22} / (1.77 \times 10^6 + 400000)} = 1503.061392088526$$

$$E_k =$$

$$0.5 \times 50 \times 1503.061392088526 \times 1503.061392088526$$

$$9 = 5.6479838709677 \times 10^7$$

$$5.65 \times 10^7 \text{J}$$

c) total energy?

$$E_{\text{total}} = 1/2 E_g = -5.65 \times 10^7 \text{J}$$

d) escape speed from the surface of the moon.

$$E_g + E_k = 0$$

$$-GMm/r + 1/2 mv^2 =$$

$$v = \sqrt{2GM/r} =$$

$$\sqrt{2 \times 6.67 \times 10^{-11} \times 7.35 \times 10^{22} / 1.74 \times 10^6} =$$

$$2,373.815494093844$$

$$2.4 \text{ km/s}$$

e) if you hit a golf ball from the surface of the moon at 1700 m/s, how high would it go?

total energy is conserved

$$E_{gi} + E_{ki} = E_{gf} + E_{kf}$$

$$-GMm/r + 1/2mv^2 = -GMm/r_f + 0$$

$$-6.67 \times 10^{-11} \times 7.35 \times 10^{22} / 1.74 \times 10^6 = -2.8175 \times 10^6$$

$$0.5 \times (1700) \times 1700 = 1,445,000$$

$$1,445,000 - 2.8175 \times 10^6 = -1,372,500 = -GM/r_f$$

$$r_f = -6.67 \times 10^{-11} \times 7.35 \times 10^{22} / -1,372,500 = 3.5719 \times 10^6$$

$$\text{final radius} = 3.57 \times 10^6 \text{ m}$$

or final height of $3.5719 \times 10^6 - 1.74 \times 10^6 = 1.8319 \times 10^6$
 of 1.83×10^6 m off the surface of the moon.

